

The effect of the three-spin interaction and the next-nearest neighbor interaction on the quenching dynamics of a transverse Ising model

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We study the zero temperature quenching dynamics of various extensions of the transverse Ising model (TIM) when the transverse field is linearly quenched from $-\infty$ to $+\infty$ (or zero) at a finite and uniform rate. The rate of quenching is dictated by a characteristic scale given by τ . The density of kinks produced in these extended models while crossing the quantum critical points during the quenching process is calculated using a many body generalization of the Landau-Zener transition theory. The density of kinks in the final state is found to decay as $\tau^{-1/2}$. In the first model considered here, the transverse Ising Hamiltonian includes an additional ferromagnetic three spin interaction term of strength J_3 . For $J_3 < 0.5$, the kink density is found to increase monotonically with J_3 whereas it decreases with J_3 for $J_3 > 0.5$. The point $J_3 = 0.5$ and the transverse field $h = -0.5$ is multicritical where the density shows a slower decay given by $\tau^{-1/6}$. We also study the effect of ferromagnetic or antiferromagnetic next nearest neighbor (NNN) interactions on the dynamics of TIM under the same quenching scheme. In a mean field approximation, the transverse Ising Hamiltonians with NNN interactions are identical to the three spin Hamiltonian. The NNN interactions non-trivially modifies the dynamical behavior, for example an antiferromagnetic NNN interactions results to a larger number of kinks in the final state in comparison to the case when the NNN interaction is ferromagnetic.

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I. INTRODUCTION

The critical dynamics of classical systems have been studied extensively in last three decades while the study of the dynamics of a quantum system when swept across a quantum critical point (QCP) is fairly recent and not yet fully understood. The vanishing of energy gap between the ground state and the first excited state of the quantum Hamiltonian signals the existence of a QCP^{1,2}. At a QCP, the correlation length as well as the relaxation time diverge, a phenomenon known as the critical slowing down. This diverging timescale makes it impossible for any system to cross the quantum critical point without excitations from the ground state. The dynamics therefore is non-adiabatic in contrast to an adiabatic evolution where the system sticks to the instantaneous ground state through out the quenching process. In recent years, there has been an upsurge in the study of dynamics close to a quantum critical point clearly indicating a growing interest in the field^{3,4,5,6,7,8,9,10,11,12,13}.

One of such attempts was to extend the Kibble's theory of defect production introduced to explain early universe behavior¹⁴ to the second order quantum phase transitions. This method of calculating the density of defects is known as the Kibble-Zurek mechanism (KZM)¹⁵. The theory of KZM for a classical second order phase transition is based on the universality of the critical slowing down and leads to the prediction that the linear dimension of the ordered domains scales with the transition time τ as τ^w where w is some combination of critical exponents. KZM has been confirmed by numerical simulations of time dependent Ginzburg-Landau model¹⁶ and also for various experimental systems¹⁷. On the other

hand, for He-4 superfluid transition, KZM could not be verified experimentally¹⁸. Hence more experiments are clearly needed to put KZM theory on a stronger footing. The same idea has been applied to study the dynamics across a zero temperature QCP by different groups^{3,4,5,6,7,8,9,10}.

The extended KZM for the zero-temperature quantum transitions relies on the fact that during the evolution when the system is close to the static critical point, the relaxation time diverges in a power-law fashion. The non-adiabatic effects become prominent when the time scale associated with the change of the Hamiltonian is of the order of the relaxation time. The loss of adiabaticity while crossing a quantum critical point can be quantified by estimating either the density of defects (e.g., the density of oppositely oriented spins in Ising models) in the final state^{3,4,5,6,7} or the fidelity of the final state with respect to the ground state³ or the residual energy^{19,20,21,22,23,24}. The argument given above immediately leads to a $(1/\sqrt{\tau})$ -dependence of the density of defects on the characteristic timescale τ of the quenching. The residual energy is defined as the difference between the energy of the evolved ground state and the true ground state. This residual energy for the integrable disorder free systems is trivially proportional to the density of kinks with the proportionality constant being equal to the strength of interaction. In an optimization approach popularly known as the "quantum annealing"^{19,20,21,22,23,24}, the strength of the quantum fluctuations is slowly reduced to zero so that a disordered and frustrated system of finite size is expected to reach adiabatically its true classical ground state. In the present literature, the expressions "annealing" and "quenching" are used synonymously. The residual en-

ergy turns out to be a more appropriate measure of non-adiabaticity for the annealing approach. In a recent work by Caneva *et al.*²⁵, it has been shown that for a disordered quantum Ising spin chain, the residual energy and the density of kinks show different scaling behavior with τ . Recently a general analysis has been carried out of the breakdown of the adiabatic limit in low-dimensional gapless systems²⁶.

In this paper, we will concentrate on the estimation of density of defects produced during the dynamics of three different types of model Hamiltonians, all of them being exactly solved, at least in a mean field level, via the Jordan Wigner transformation. These three Hamiltonians are extensions of the TIM with an additional interaction term in each and our aim is to study the effect of such interactions on the density of defects produced during the quenching. The additional terms are i) a ferromagnetic three spin interaction²⁷ ii) an antiferromagnetic next nearest neighbor interaction and iii) a ferromagnetic next nearest neighbor interaction, respectively. We consider the unitary evolution of the system prepared in the ground state of the initial Hamiltonian which crosses its equilibrium critical line as the system evolves. As described later, in all the cases, the fermionization of the Hamiltonian reduces it to a quadratic form and hence one can reduce the dynamics of a many-body Hamiltonian effectively to a 2×2 Landau Zener problem²⁸ in the fourier representation.

The paper is organized as follows. Section II includes a detailed discussion on the analytical diagonalization of the transverse Ising Hamiltonian with a three spin interaction term. In section III, we have described the transverse quenching scheme along with the results for the above model. We have presented a comparison between the three spin Hamiltonian and the Hamiltonians with next nearest neighbor interactions when treated at a mean field level in section IV. A brief summary of the work is presented in the concluding section with a brief discussion based on the recent developments in this field.

II. MODEL AND THE PHASE DIAGRAM

The Hamiltonian of a one-dimensional three spin interacting transverse Ising system is given by²⁷

$$H = -\frac{1}{2} \left\{ \sum_i \sigma_i^z [h + J_3 \sigma_{i-1}^x \sigma_{i+1}^x] + J_x \sum_i \sigma_i^x \sigma_{i+1}^x \right\}, \quad (1)$$

where σ^z and σ^x are non-commuting Pauli spin matrices, J_x is the strength of the nearest neighbor ferromagnetic interaction while J_3 denotes the strength of the three spin interaction. In the limit $J_3 \rightarrow 0$, the model reduces to the celebrated transverse Ising model studied extensively in recent years^{1,2}. By a duality transformation³¹, the above Hamiltonian can be mapped to a transverse XY model with competing (ferro-antiferro) interactions

in the x and y components of the spin²⁷. Interestingly, even in the presence of the three spin interaction term, the Hamiltonian given by Eq. (1) is exactly solved by the Jordan-Wigner (JW) transformation^{29,30,31} which maps this interacting spin system to a system of noninteracting spinless fermions. Moreover, this three spin term is found to be irrelevant in determining the quantum critical behavior of the system. The critical exponents are the same as that of Ising model in a transverse field except for the case $J_x = 0$, and $J_3 = h$. For the sake of completeness, let us now provide a brief discussion on the diagonalization of the Hamiltonian.

In the JW-transformation, the Pauli matrices are transformed to a set of fermionic operators (c_i) defined as

$$c_i = \sigma_i^- \exp(-i\pi \sum_{j=1}^{i-1} \sigma_j^\dagger \sigma_j^-) \\ \sigma_i^z = 2c_i^\dagger c_i - 1 \quad (2)$$

with $\sigma^\dagger = (\sigma^x + i\sigma^y)/2$ and $\sigma^- = (\sigma^x - i\sigma^y)/2$, and satisfy the standard anticommutation relations

$$\{c_i^\dagger, c_j\} = \delta_{ij}, \quad \{c_i^\dagger, c_j^\dagger\} = \{c_i, c_j\} = 0.$$

We shall work in the basis in which σ^z is diagonal so that the presence of a fermion at a particular site i corresponds to an up spin (i.e., eigenvalue $+1$ of the operator σ_i^z) at that site. Using a periodic boundary condition, the Fourier transform of the Hamiltonian can be cast in the form

$$H = - \left[\sum_{k>0} (h + \cos k - J_3 \cos 2k) (c_k^\dagger c_k + c_{-k}^\dagger c_{-k}) + i(\sin k - J_3 \sin 2k) (c_k^\dagger c_{-k}^\dagger + c_k c_{-k}) \right]. \quad (3)$$

Clearly, in the momentum representation of c-fermions, the Hamiltonian is quadratic and is translationally invariant. Using the Bogoliubov transformation, the Hamiltonian can be diagonalized to the form $-\sum_k \epsilon_k \eta_k^\dagger \eta_k$ where η_k are the Bogoliubov quasiparticles and ϵ_k is the excitation energy or gap given by^{27,29}

$$\epsilon_k = (h^2 + 1 + J_3^2 + 2h \cos k - 2h J_3 \cos 2k - 2J_3 \cos k)^{1/2} \quad (4)$$

with J_x set equal to unity.

It can be easily shown that the gap of the spectrum vanishes at $h = J_3 + 1$ and also at $h = J_3 - 1$ with ordering (or mode-softening) wave vectors π and 0 respectively. These two lines correspond to quantum phase transitions from a ferromagnetically ordered phase to a quantum paramagnetic phase with the associated exponents being the same as the transverse Ising model³⁰. The wave vector at which the minima of ϵ_k (Eq. 4) occurs, gets shifted from $k = 0$ to $k = \pi$ wave vector when one crosses the line $h = J_3$. Moreover, there is an additional phase transition at $h = -J_3$. This transition belongs to the universality class of the anisotropic transition observed in the transverse XY-model dual to the

Hamiltonian (1)³² and the phase boundary is flanked by the incommensurate phases on either side with ordering wave vector given by

$$\cos k = \frac{h - J_3}{4hJ_3}. \quad (5)$$

This incommensurate wave vector picks up a value k_o such that $\cos k_o = 1/2J_3$ at the phase boundary. Obviously, for $J_3 < 0.5$, the anisotropic phase transition can not occur. The equilibrium phase diagram of the model is shown in Fig. 1.

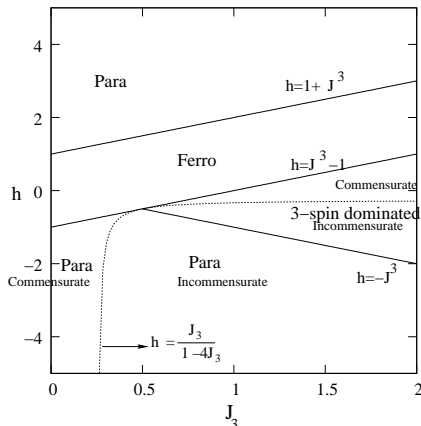


FIG. 1: Equilibrium phase diagram of the three spin interacting Ising model. Solid lines show phase boundaries and dotted line marks the boundary between the incommensurate and the commensurate phase.

III. TRANSVERSE QUENCHING AND RESULTS

The dynamics of the three spin interacting TIM is found to be very interesting due to the fact that the system crosses various quantum critical lines during the process of dynamics. As mentioned already, the system deviates from the adiabatic evolution in the neighborhood of a quantum critical point where nonadiabaticity dominates due to the divergence of relaxation time. We shall introduce the time dependence in the Hamiltonian through the transverse field which is linearly quenched from $-\infty$ to $+\infty$ at a steady finite rate given by $h(t) \sim t/\tau$, where the quenching time τ determines the rate of quenching^{3,4,5}. At time $t = -\infty$ the transverse field $h = -\infty$ and hence all the spins are pointing in the negative z -direction. By virtue of the duality transformation, the transverse quenching of the 3 spin Hamiltonian corresponds to the anisotropic quenching of the transverse XY model where the interaction term of the later Hamiltonian is adiabatically changed from $-\infty$ to ∞ ¹⁰.

Let us recall the Hamiltonian given in Eq. 3 with a time dependent transverse field $h(t)$. This Hamiltonian can be split into a sum of independent terms,

$H(t) = [\sum_{k>0} H_k(t)]$ where each $H_k(t)$ operates on a four dimensional Hilbert space spanned by the basis vectors $|0\rangle$, $|k, -k\rangle$, $|k\rangle$ and $|-k\rangle$. The vacuum state where no c -particle is present, is denoted by $|0\rangle$ which corresponds to a spin configuration with all spins pointing in the $-z$ -direction. The form of the Hamiltonian readily suggests that the parity (even or odd) of total number of fermions (given by $n_k = c_k^\dagger c_k + c_{-k}^\dagger c_{-k}$) for each mode is conserved. Therefore, to study the quenching dynamics, it is convenient to project the Hamiltonian $H_k(t)$ in the subspace spanned by $|0\rangle$ and $|k, -k\rangle$. The projected Hamiltonian has a form

$$\begin{bmatrix} h(t) + \cos k - J_3 \cos 2k & i(\sin k - J_3 \sin 2k) \\ -i(\sin k - J_3 \sin 2k) & -(h(t) + \cos k - J_3 \cos 2k) \end{bmatrix}$$

In the reduced Hilbert space, any general state can be represented as a superposition of $|0\rangle$ and $|k, -k\rangle$ with time dependent amplitudes $u_k(t)$ and $v_k(t)$ such that $\psi_k(t) = u_k(t)|0\rangle + v_k(t)|k, -k\rangle$. The time evolution of the state is given by the Schroedinger equation

$$i\partial_t \psi_k(t) = H_k(t) \psi_k(t). \quad (6)$$

We shall here use the initial conditions $u_k(-\infty) = 1$ and $v_k(-\infty) = 0$ which in the spin language corresponds to the state with all spins down. The off diagonal term $\Delta = \sin k - J_3 \sin 2k$ represents the interaction between the two time dependent levels with energies $E_{1,2} = \pm[h(t) + \cos(k) - J_3 \cos 2k]$. The zeroes of the off-diagonal term yield the mode softening wave vectors $k = 0, \pi$ and $\cos^{-1} 1/(2J_3)$ (provided $J_3 > 0.5$) at which the system becomes quantum critical for appropriate parameter values. At these parameter values and wave vectors, the system undergoes a nonadiabatic transition from its instantaneous ground state. A measure of nonadiabaticity can be obtained by comparing the two level problem to the corresponding Landau-Zener transition equations^{5,7}. For a completely adiabatic transition, we expect the final state to be described by the probability amplitudes $u_k(+\infty) = 0$ and $v_k(+\infty) = 1$, i.e., the complete spin-flip from down to up occurs. The nonadiabatic transition probability p_k is directly given by $|u_k(+\infty)|^2$ where the probability amplitudes $u_k(t)$ and $v_k(t)$ are normalized at each instant of time. Equivalently, p_k also measures the probability that the system remains in its initial state $|0\rangle$ even at the final time. Using the results of Landau-Zener transitions^{21,28}, p_k is found to be

$$p_k = |u_k(+\infty)|^2 = \exp(-2\pi\gamma) \text{ where } \gamma = \frac{\Delta^2}{\frac{d}{dt}(E_1 - E_2)}. \quad (7)$$

Therefore, in this model

$$p_k = \exp[-\pi\tau(\sin k - J_3 \sin 2k)^2]. \quad (8)$$

The variation of p_k as a function of k for different values of quenching time τ is shown in Fig. 2. It is to be noted that for $J_3 < 0.5$, there are peaks at $-\pi, 0$ and π in the whole range of wave vectors from $-\pi$ to π whereas for

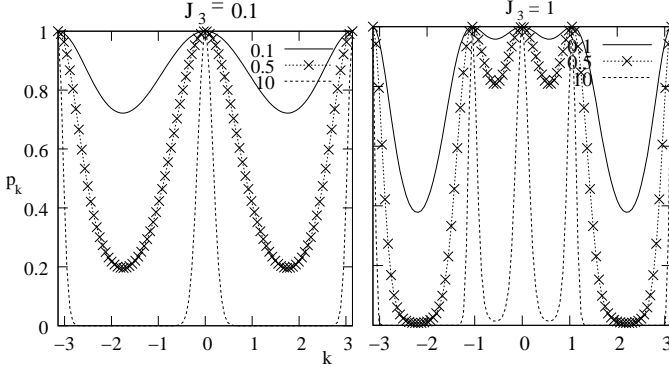


FIG. 2: Non-adiabatic transition probability p_k for the three spin interacting Hamiltonian with $J_3 = 0.1$ in Fig. 2(a) and $J_3 = 1$ in Fig. 2(b) for various τ . It should be noted that for $J_3 = 1$, the system undergoes a non-adiabatic transition at an incommensurate wave vector $k = \pi/3$ and therefore, there is an additional peak at this wave vector. For large τ , p_k is nonzero only for wave vectors very close to the critical modes. On the other hand, for small values of τ , levels cross quickly resulting to a non-zero value of p_k for all values of k .

$J_3 > 0.5$ there are additional peaks at the incommensurate values $\pm \cos^{-1}(1/2J_3)$.

As mentioned already, the degree of nonadiabaticity can be quantified through the density of kinks n generated at $t = +\infty$ which is obtained by integrating the probability p_k over the entire range of wave vector.

$$n = \sum_k p_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk p_k. \quad (9)$$

A close inspection of Eq. 8 (see also Fig. 2) shows that for sufficiently slow quenching (i.e., large τ), only modes close to the critical modes are excited. One can therefore, to the lowest order in k , replace $\sin k$ by k in the exponential of Eq. 8 and arrive at an approximate analytical expression for density of kinks in the large τ limit, given by:

$$n = \frac{1}{2\pi(1-2J_3)\sqrt{\tau}} + \frac{1}{2\pi(1+2J_3)\sqrt{\tau}} \quad (\text{for } J_3 < 0.5) \quad (10a)$$

$$n = \frac{2}{2\pi(2J_3-1)\sqrt{\tau}} + \frac{2}{2\pi(1+2J_3)\sqrt{\tau}} \quad (\text{for } J_3 > 0.5). \quad (10b)$$

In Eq. 10(a), the first term corresponds to the contribution from modes close to $k = 0$ whereas the second term is due to the peaks at $k = \pi, -\pi$. For the case $J_3 > 0.5$, the contribution from peaks at $k = 0, \pi$ and $-\pi$ happens to be the same as Eq. 10(a) whereas the contribution n_1 from the modes close to $k = k_0$ is also equal to 10(a) in the following way:

$$\begin{aligned} n_1 &= \frac{1}{\pi} \int_0^{\pi} \exp[-\pi\tau\{(\cos k_0 - 2J_3 \cos 2k_0)(k-k_0)\}^2] \\ &= \frac{1}{2\pi\sqrt{\tau}} \left[\frac{1}{2J_3+1} + \frac{1}{2J_3-1} \right]. \end{aligned} \quad (11)$$

The density of kinks monotonically increases with increasing J_3 provided $J_3 < 0.5$ because of the decrease in the off-diagonal term making the probability of excitations higher. On the other hand, for $J_3 > 0.5$, the off-diagonal term monotonically increases with increasing J_3 resulting to an overall decrease in the density of kinks, see figure 3. These results can also be seen from the approximate analytical expression of the kink density given in Eq. (10 a and b) for both the cases.

We shall now focus our attention to the case $J_3 = 0.5$. In the process of the transverse quenching, the system crosses the multicritical point at $h = -0.5, J_3 = 0.5$ as shown in the Fig. 1 and a special power-law behavior of the kink density is observed at these parameter values. The transition probability p_k maximizes at $k = 0$ as shown above. The argument of the exponential in p_k is expanded about $k=0$ at $J_3 = 0.5$, leading to a form $p_k = \exp[-\pi\tau k^6/4]$. The contribution to the kink density scales as $1/\tau^{1/6}$ which can be obtained by simply integrating this p_k , see figure 3. This relatively slow decay of density is a special characteristic of quenching through a multicritical point. A similar behavior is also seen in the anisotropic quenching of the transverse XY model¹⁰.

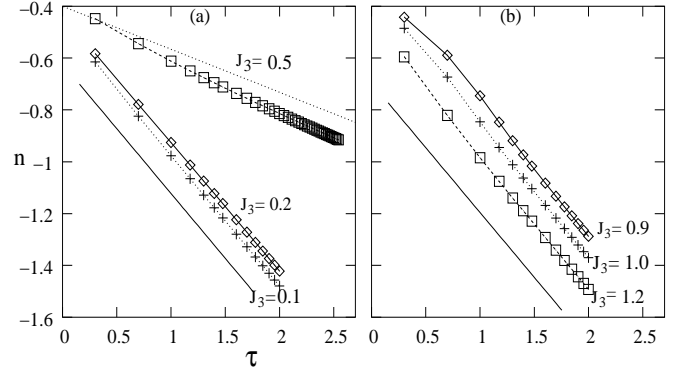


FIG. 3: The variation of kink density with τ for different J_3 (< 0.5) is shown in Fig. 3a. Kink density increases with J_3 . On the other hand for $J_3 > 0.5$, this variation decreases with J_3 as shown in Fig. 3b. The thick line has slope -0.5 and the slope of the dotted line is $-1/6$, a behavior observed at $J_3 = 0.5$

One can also study the effect of the anisotropic quenching which involves quenching of the nearest neighbor Ising interaction term $J^x(t) (\sim t/\tau)$ from $-\infty$ to $+\infty$ instead of the transverse field with the three spin interaction term set to unity. At $t \rightarrow -\infty$, the ground state of the system is antiferromagnetic along x . The probability of the non-adiabatic transition is similarly given by

$$p_k = \exp[-\pi\tau\{(h+1)\sin k\}^2]. \quad (12)$$

It is interesting to note that for $h = -1$, p_k is unity for all values of k . The density of kinks for the anisotropic quenching is given as

$$n = \frac{1}{2\pi} \int dk p_k = \exp((-\pi\tau(h+1)\sin k)^2) \quad (13)$$

with an approximate analytical form given as

$$n = \frac{1}{\pi(h+1)\sqrt{\tau}} \quad (14)$$

which shows that the density of kinks decreases monotonically with h . This can be attributed to an increase in the off-diagonal term of the Hamiltonian.

IV. CONNECTION TO THE TRANSVERSE QUENCHING OF THE MODELS WITH NEXT NEAREST NEIGHBOR INTERACTIONS

We shall now use the results of the previous section to study the transverse quenching of a quantum Ising model with uniform ferromagnetic nearest neighbor interaction and also an additional NNN interaction which is either antiferromagnetic or ferromagnetic. The model with NNN antiferromagnetic interaction has regular frustrations and is popularly known as Axial Next Nearest Neighbor Ising (ANNNI) model³³ in a transverse field. We shall show below that within a mean field approximation, the three spin model has a close resemblance to the one-dimensional NNN interacting TIMs.

The Hamiltonian of the transverse ANNNI model is given by

$$H = -\frac{1}{2} \left\{ \sum_i^N [h\sigma_i^z + J_1\sigma_i^x\sigma_{i+1}^x - J_2\sigma_i^x\sigma_{i+2}^x] \right\} \quad (15)$$

where $J_1, J_2 > 0$. Henceforth, without loss of any generality, we shall set $J_1 = 1$. At $h = 0$, the ground state is ferromagnetically ordered for $J_2 < 0.5$, whereas the system shows an “anti-phase” ordering (where two up spins are followed by two down spins) for $J_2 > 0.5$. The two phases meet at an infinitely degenerate multi-critical point $J_2 = 0.5$ and $h = 0$. The quantum fluctuations introduced by the transverse field h competes with the ferromagnetic (or the antiphase) order and eventually the system undergoes a quantum phase transition to a paramagnetic phase at a critical value of the transverse field given by h_c which is a function of the NNN interaction J_2 . One-dimensional quantum ANNNI model shows a rich phase diagram which is not fully understood till date^{2,33,34}.

When mapped to the corresponding fermionic Hamiltonian via a JW transformation, the NNN interaction leads to a four-fermion term in the fermionic version of the Hamiltonian. In the limit $J_2 \rightarrow 0$, this term vanishes so that the model is exactly solvable in terms of non-interacting fermions. For non-zero J_2 , the fermionic Hamiltonian is written as

$$H = -\frac{1}{2} \left\{ \left[\sum_i h(2c_i^\dagger c_i - 1) + (c_i^\dagger - c_i)(c_{i+1}^\dagger + c_{i+1}) - J_2(c_i^\dagger - c_i)(1 - 2c_{i+1}^\dagger c_{i+1})(c_{i+2}^\dagger + c_{i+2}) \right] \right\}. \quad (16)$$

The occurrence of the four-fermion term renders the model analytically intractable though an approximate

analytical solution is possible at least in the limit of small J_2 . Deep in the paramagnetic phase all the spins are oriented in the direction of the transverse field so that $\langle \sigma_i^z \rangle = 1$ or in the fermionic language $1 - 2c_{i+1}^\dagger c_{i+1} = -1$. We shall approximate $\langle \sigma_i^z \rangle = 1$ for all positive values of h including $h \sim 0$. This approximation, though crude, transforms the four fermion term $(c_i^\dagger - c_i)(1 - 2c_{i+1}^\dagger c_{i+1})(c_{i+2}^\dagger + c_{i+2})$ to a quadratic form. The Hamiltonian becomes exactly solvable but the rich phase diagram of the model is not captured in this approximation^{35,36}. Within this approximation, we shall explore the role of small NNN antiferromagnetic interaction on the density of kinks produced during the transverse quenching. As described below, this approximation at least shows a decrease of critical field h_c with J_2 for $J_2 < 0.5$.

The mean field Hamiltonian in the momentum space is

$$H = - \left[\sum_{k>0} (h + \cos k + J_2 \cos 2k)(c_k^\dagger c_k + c_{-k}^\dagger c_{-k}) + i(\sin k + J_2 \sin 2k)(c_k^\dagger c_{-k}^\dagger + c_k c_{-k}) \right] \quad (17)$$

Comparing Eq. (17) with Eq. (3), one finds that the transverse ANNNI chain Hamiltonian in the mean field approximation is identical to the three spin Hamiltonian if the antiferromagnetic interaction J_2 of the former is replaced by the negative of the three spin interaction term (J_3) in the latter. Using the results of the previous section, the phase diagram of the mean field ANNNI model can be found out for $h > 0$ (see Fig. 4). The phase boundary between the ferromagnetic phase and the paramagnetic is given by $h = 1 - J_2$ with an ordering wave vector π (this corresponds to the Ising transition at $h = J_3 + 1$ of Fig. 1). For $J_3 > 0.5$, i.e., the transition between the antiphase and the paramagnetic phase, is given by the corresponding anisotropic transition of three spin model with the phase boundary given by the equation $h = J_2$ and the ordering wave vector has an incommensurate value as given in Eq. 5.

The approximation $1 - 2c_i^\dagger c_i = -1$ is valid for positive h only, we choose a quenching scheme where the transverse field has a functional dependence $h(t) \sim -t/\tau$ with t going from $-\infty$ to 0 so that $h(t)$ remains positive for the entire quenching period and vanishes at the end of the quenching. Therefore the system does not cross the Ising critical line $-h = J_2 + 1$.

In the final state at $t = 0$, all the spins are expected to orient in the x -direction with a ferromagnetic order. The density of oppositely oriented spins at $t \rightarrow 0$ is related to J_2 as

$$n = \frac{1}{2\pi(1 - 2J_2)\sqrt{\tau}} \quad (18)$$

which shows that the density of kinks increases monotonically with J_2 .

It should be noted that if we follow a quenching scheme in which the transverse field is changed from $-\infty$ to zero,

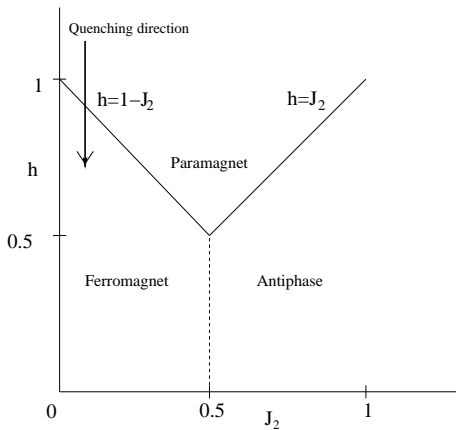


FIG. 4: Mean field phase diagram of the ANNNI model in the $h - J_2$ plane. We study the quenching dynamics across the phase boundary close to $J_2 \rightarrow 0$.

we must approximate the term $1 - 2c_{i+1}^\dagger c_{i+1}$ with $+1$ rather than -1 for above calculations to be viable. In the process of dynamics, the system crosses the quantum critical line $-h = J_2 + 1$ with the modes close to $k = 0$ getting excited. This approach as well leads to identical result for kink density (as given in Eq. 18). Therefore, the presence of a small antiferromagnetic NNN interaction adds to the kink-production in comparison to the ferromagnetic transverse Ising model ($J_2 = 0$) with the same quenching scheme.

One can also study, in the similar spirit, a model with a small ferromagnetic NNN interaction J_{FM} . We use the same mean field approximation for $h \geq 0$ so that this model is identical to the three spin model with $J_3 = J_{FM}$. A similar calculation leads once again to a $1/\sqrt{\tau}$ fall of the density of kinks given by

$$n = \frac{1}{2\pi(1 + 2J_{FM})\sqrt{\tau}}. \quad (19)$$

This is expected because the NNN ferromagnetic interaction enhances the strength of the ferromagnetic ordering and hence the probability of excitations or density of kinks is lowered.

V. CONCLUSIONS

In this paper, we have studied the effect of various additional interactions on the dynamics of the transverse Ising model when swept across the quantum critical lines. The defect density scale with the timescale τ as $\tau^{-1/2}$, like in transverse Ising case, with a prefactor which varies

from model to model. The first of the variants includes a three spin interaction with strength J_3 . Here, the phase diagram indicates the existence of an anisotropic phase transition at an incommensurate value of wave vector in addition to the normal Ising transition for $J_3 > 0.5$. Interestingly, we observe that the density of kinks increases monotonically with J_3 for $J_3 < 0.5$ whereas decreases for $J_3 > 0.5$. On the other hand, at $J_3 = 0.5$, the contribution to the kink density scales as $\tau^{-1/6}$ due to the existence of a multicritical point at $J_3 = 0.5$. The other set of Hamiltonians include a ferromagnetic or an antiferromagnetic next nearest neighbor interactions. The presence of the four fermion term makes such a Hamiltonian analytically intractable. We have used a mean field approximation to reduce the four fermion term in the fermionized representation to a quadratic term. The quenching scheme is chosen carefully so that the regions where the approximation is not valid are avoided in the process of dynamics. Using the similarity between the fermionized next nearest neighbor interacting Hamiltonians under the mean field approximation, and the three spin interacting model, the density of kink in the final state is estimated. It is observed that the ferromagnetic next nearest neighbor interactions reduces the density of kinks produced as opposed to the case of antiferromagnetic next nearest neighbor interaction because such a ferromagnetic interaction enhances the ferro-ordering discouraging the production of kinks. On the other hand, frustration leads an enhanced non-adiabatic transitions. We should mention in conclusion that it is in principle possible to construct a better mean field theory for the ANNNI model³⁶, however, no qualitative change in the dynamical behaviour in the region $J_2 \rightarrow 0$ is expected.

We conclude with the comment that the models studied in the present work are integrable (at least in the mean field limit) which leads to a $1/\sqrt{\tau}$ scaling behavior of the defect density. However, in a random or a non-integrable system such a behavior need not be expected²⁵. The quenching and annealing dynamics of several non-integrable systems along with the dependence of the defect density on the integrability of the model are yet to be completely understood. We have also observed a much slower decay of the form $1/\tau^{1/6}$ when quenched through the multicritical point of the three spin model as in the anisotropic quenching of the transverse XY chain¹⁰.

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